Mass continuity equation in the electromagnetic field

Ying Weng[1], **Zi-Hua Weng**[2]

[1] College of Chemistry & Chemical Engineering,
Xiamen University, Xiamen 361005, China
[2] School of Physics and Mechanical & Electrical Engineering,
Xiamen University, Xiamen 361005, China

Abstract— A theoretical method with the quaternion algebra was presented to derive the mass continuity equation from the linear momentum. It predicts that the strength of electromagnetic field and the velocity have the impact on the mass continuity equation. In the gravitational field and electromagnetic field, the mass continuity equation will change with the electromagnetic field strength, gravitational field strength, linear momentum, electric current, and the speed of light. The deduction can explain why the field strength has an influence on the anomalous transport about the mass continuity equation in the plasma and electrolytes etc.

1. INTRODUCTION

The conservation laws are important invariants for the electromagnetic field. In the electromagnetic field theory described by the vectorial quantity, the mass continuity equation is solely dealt with the mass rate and the divergence of linear momentum. However, this opinion can not explain why the field strength has an impact on the anomalous transport about the mass continuity equation in the plasma and electrolytes [1, 2].

The algebra of quaternion [3] was first used by J. C. Maxwell [4] to describe the property of electromagnetic field. Similarly, the quaternion can also be used to demonstrate the feature of gravitational field, although these two fields are quite different [5]. By means of the scalar invariant of quaternions, we find that the mass continuity equation is an invariant in the gravitational field and electromagnetic field. In the quaternion spaces, the definition of mass continuity equation can be extended to the case for coexistence of electromagnetic field and gravitational field.

Presently, the mass continuity equation [6] was limited to the case of weak gravitational strength. It is found out that all of related verifications are solely constrained to be in the range of weak field strengths, and have not been validated in the strong fields up to now. With the characteristics of octonions [7], we find some impact factors of the mass continuity equation. In the electromagnetic field and gravitational field, the velocity and the strengths of electromagnetic field and gravitational field have a few influences on the mass continuity equation.

The results state that the gravitational field strength and electromagnetic field strength have an influence on the mass continuity equation, although the impact of the field strengths are usually very tiny when the electromagnetic field and gravitational field both are weak. And then, the mass continuity equation is conserved in most cases. However, when the electromagnetic field and gravitational field are strong enough, their field strengths will affect the mass continuity equation obviously, and cause the anomalous transport in the plasma.

2. COORDINATES TRANSFORMATION

In the quaternion space, the basis vector for the gravitational field is $\mathbb{E}_g = (1,\, \boldsymbol{i}_1,\, \boldsymbol{i}_2,\, \boldsymbol{i}_3)$, and that for the electromagnetic field is $\mathbb{E}_e = (\boldsymbol{I}_0,\, \boldsymbol{I}_1,\, \boldsymbol{I}_2,\, \boldsymbol{I}_3)$. The $\mathbb{E}_e$ is independent of the $\mathbb{E}_g$, with $\mathbb{E}_e = \mathbb{E}_g \circ \boldsymbol{I}_0$. The basis vectors $\mathbb{E}_g$ and $\mathbb{E}_e$ can be combined together to become the basis vector $\mathbb{E}$ of the octonion space.

$$\mathbb{E} = (1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3, \boldsymbol{I}_0, \boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3) \tag{1}$$

The octonion quantity $\mathbb{D}(d_0, d_1, d_2, d_3, D_0, D_1, D_2, D_3)$ is defined as follows.

$$\mathbb{D} = d_0 + \Sigma(d_j \boldsymbol{i}_j) + \Sigma(D_i \boldsymbol{I}_i) \tag{2}$$

where, d_i and D_i are all real; $i = 0, 1, 2, 3$; $j = 1, 2, 3$.

When the coordinate system is transformed into the other one, the physical quantity $\mathbb{D}$ will be become the octonion physical quantity $\mathbb{D}'(d'_0, d'_1, d'_2, d'_3, D'_0, D'_1, D'_2, D'_3)$.

$$\mathbb{D}' = \mathbb{K}^* \circ \mathbb{D} \circ \mathbb{K} \tag{3}$$

where, $\mathbb{K}$ is the octonion, and $\mathbb{K}^* \circ \mathbb{K} = 1$; $*$ denotes the conjugate of octonion; $\circ$ is the octonion multiplication.

In case of the d_0 does not take part in the coordinates transformation in the above, we have,

$$d_0 = d'_0 \ , \ \mathbb{D}^* \circ \mathbb{D} = (\mathbb{D}')^* \circ \mathbb{D}' \ . \tag{4}$$

In the above equation, the scalar part is one and the same during the octonion coordinates are transformed. Some invariants including the mass continuity equation about the electromagnetic field will be obtained from this characteristics of octonions.

Table 1: The octonion multiplication table.

	1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$	$\boldsymbol{I}_0$	$\boldsymbol{I}_1$	$\boldsymbol{I}_2$	$\boldsymbol{I}_3$
1	1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$	$\boldsymbol{I}_0$	$\boldsymbol{I}_1$	$\boldsymbol{I}_2$	$\boldsymbol{I}_3$
$\boldsymbol{i}_1$	$\boldsymbol{i}_1$	-1	$\boldsymbol{i}_3$	$-\boldsymbol{i}_2$	$\boldsymbol{I}_1$	$-\boldsymbol{I}_0$	$-\boldsymbol{I}_3$	$\boldsymbol{I}_2$
$\boldsymbol{i}_2$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_3$	-1	$\boldsymbol{i}_1$	$\boldsymbol{I}_2$	$\boldsymbol{I}_3$	$-\boldsymbol{I}_0$	$-\boldsymbol{I}_1$
$\boldsymbol{i}_3$	$\boldsymbol{i}_3$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_1$	-1	$\boldsymbol{I}_3$	$-\boldsymbol{I}_2$	$\boldsymbol{I}_1$	$-\boldsymbol{I}_0$
$\boldsymbol{I}_0$	$\boldsymbol{I}_0$	$-\boldsymbol{I}_1$	$-\boldsymbol{I}_2$	$-\boldsymbol{I}_3$	-1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$
$\boldsymbol{I}_1$	$\boldsymbol{I}_1$	$\boldsymbol{I}_0$	$-\boldsymbol{I}_3$	$\boldsymbol{I}_2$	$-\boldsymbol{i}_1$	-1	$-\boldsymbol{i}_3$	$\boldsymbol{i}_2$
$\boldsymbol{I}_2$	$\boldsymbol{I}_2$	$\boldsymbol{I}_3$	$\boldsymbol{I}_0$	$-\boldsymbol{I}_1$	$-\boldsymbol{i}_2$	$\boldsymbol{i}_3$	-1	$-\boldsymbol{i}_1$
$\boldsymbol{I}_3$	$\boldsymbol{I}_3$	$-\boldsymbol{I}_2$	$\boldsymbol{I}_1$	$\boldsymbol{I}_0$	$-\boldsymbol{i}_3$	$-\boldsymbol{i}_2$	$\boldsymbol{i}_1$	-1

3. VELOCITY

The radius vector is $\mathbb{R}_g = (r_0,\ r_1,\ r_2,\ r_3)$ in the quaternion space for gravitational field. For the electromagnetic field, the radius vector is $\mathbb{R}_e = (R_0,\ R_1,\ R_2,\ R_3)$. Their combination is the radius vector $\mathbb{R}(r_0, r_1, r_2, r_3, R_0, R_1, R_2, R_3)$ in the octonion space.

$$\mathbb{R} = r_0 + \Sigma(r_j \boldsymbol{i}_j) + \Sigma(R_i \boldsymbol{I}_i) \tag{5}$$

where, $r_0 = v_0 t$. v_0 is the speed of light, t denotes the time.

In the quaternion space for the gravitational field, the velocity is $\mathbb{V}_g = (v_0,\ v_1,\ v_2,\ v_3)$. For the electromagnetic field, the velocity is $\mathbb{V}_e = (V_0,\ V_1,\ V_2,\ V_3)$. They can be combined together to become the octonion velocity $\mathbb{V}(v_0, v_1, v_2, v_3, V_0, V_1, V_2, V_3)$ in the octonion space.

$$\mathbb{V} = v_0 + \Sigma(v_j \boldsymbol{i}_j) + \Sigma(V_i \boldsymbol{I}_i) \tag{6}$$

In some cases, the electric charge is combined with the mass to become the electron or proton etc., we have the relation $R_i \boldsymbol{I}_i = r_i \boldsymbol{i}_i \circ \boldsymbol{I}_0$ and $V_i \boldsymbol{I}_i = v_i \boldsymbol{i}_i \circ \boldsymbol{I}_0$, with $\boldsymbol{i}_0 = 1$.

4. FIELD STRENGTH

From the Aharonov-Bohm effect, we find that the field potential is much more essential than the field strength. By means of the field potential and the quaternion operator, the field strength can be defined to cover the strengths of gravitational field and electromagnetic field.

The gravitational potential is $\mathbb{A}_g = (a_0, a_1, a_2, a_3)$, and the electromagnetic potential is $\mathbb{A}_e = (A_0, A_1, A_2, A_3)$. The gravitational potential and electromagnetic potential constitute the field potential $\mathbb{A}(a_0, a_1, a_2, a_3, A_0, A_1, A_2, A_3)$ in the octonion space.

$$\mathbb{A} = \mathbb{A}_g + k_{eg} \mathbb{A}_e \tag{7}$$

where, k_{eg} is the coefficient.

The octonion strength $\mathbb{B}$ consists of gravitational strength $\mathbb{B}_g$ and electromagnetic strength $\mathbb{B}_e$.

$$\mathbb{B} = \lozenge \circ \mathbb{A} = \mathbb{B}_g + k_{eg} \mathbb{B}_e \tag{8}$$

where, the quaternion operator $\lozenge = \Sigma(\boldsymbol{i}_i \partial_i)$, with $\partial_i = \partial / \partial r_i$.

In the above equation, we choose two gauge conditions of field potential to simplify succeeding calculation. The gravitational potential gauge is $b_0 = \partial_0 a_0 + \nabla \cdot \mathbf{a} = 0$, and the electromagnetic potential gauge $B_0 = \partial_0 A_0 + \nabla \cdot \mathbf{A} = 0$. Where, $\mathbf{a} = \Sigma(a_j \boldsymbol{i}_j)$; $\mathbf{A} = \Sigma(A_j \boldsymbol{i}_j)$; $\nabla = \Sigma(\boldsymbol{i}_j \partial_j)$.

The gravitational field strength $\mathbb{B}_g = \mathbf{g}/c + \mathbf{b}$ includes two components, $\mathbf{g}/c = \partial_0 \mathbf{a} + \nabla a_0$ and $\mathbf{b} = \nabla \times \mathbf{a}$. At the same time, the electromagnetic field strength $\mathbb{B}_e = \mathbf{E}/c + \mathbf{B}$ involves two parts, $\mathbf{E}/c = (\partial_0 \mathbf{A} + \nabla A_0) \circ \boldsymbol{I}_0$ and $\mathbf{B} = -(\nabla \times \mathbf{A}) \circ \boldsymbol{I}_0$.

In the Newtonian gravitational field theory, there are $\mathbf{a} = 0$ and $\mathbf{b} = 0$ specially.

5. LINEAR MOMENTUM

The linear momentum is one function of the field source, which is a combination of the gravitational field source and electromagnetic field source.

The linear momentum density $\mathbb{S}_g = m\mathbb{V}_g$ is the source of the gravitational field, and the electric current density $\mathbb{S}_e = q\mathbb{V}_g \circ \boldsymbol{I}_0$ is that of the electromagnetic field. They combine together to become the field source $\mathbb{S}$.

$$\mu\mathbb{S} = -(\mathbb{B}/v_0 + \lozenge)^* \circ \mathbb{B} = \mu_g \mathbb{S}_g + k_{eg}\mu_e \mathbb{S}_e - \mathbb{B}^* \circ \mathbb{B}/v_0 \tag{9}$$

where, $k_{eg}^2 = \mu_g/\mu_e$; q is the electric charge density; m is the inertial mass density; μ, μ_g, and μ_e are the constants.

The $\mathbb{B}^* \circ \mathbb{B}/(2\mu_g)$ is the energy density, and includes that of the electromagnetic field.

$$\mathbb{B}^* \circ \mathbb{B}/\mu_g = \mathbb{B}_g^* \circ \mathbb{B}_g/\mu_g + \mathbb{B}_e^* \circ \mathbb{B}_e/\mu_e \tag{10}$$

The octonion linear momentum density is

$$\mathbb{P} = \mu\mathbb{S}/\mu_g = \widehat{m} v_0 + \Sigma(m v_j \boldsymbol{i}_j) + \Sigma(M V_i \boldsymbol{i}_i \circ \boldsymbol{I}_0) \tag{11}$$

where, $\widehat{m} = m + \triangle m$; $M = k_{eg}\mu_e q/\mu_g$; $\triangle m = -(\mathbb{B}^* \circ \mathbb{B}/\mu_g)/v_0^2$.

The above means that the gravitational mass density $\widehat{m}$ is changed with the strength of either electromagnetic field or gravitational field.

6. MASS CONTINUITY EQUATION

The applied force can not be covered by Maxwell's equations, which are derived from the definition of field source. Whereas, the applied force can be derived from the linear momentum. And that the applied force covers the mass continuity equation in the gravitational and electromagnetic fields.

In the octonion space, the applied force density $\mathbb{F}$ is defined from the linear momentum density $\mathbb{P}$ in the gravitational field and electromagnetic field.

$$\mathbb{F} = v_0(\mathbb{B}/v_0 + \lozenge)^* \circ \mathbb{P} \tag{12}$$

where, the applied force includes the gravity, the inertial force, the Lorentz force, and the interacting force between the magnetic strength with magnetic moment, etc.

The applied force density $\mathbb{F}$ is rewritten as follows.

$$\mathbb{F} = f_0 + \Sigma(f_j \boldsymbol{i}_j) + \Sigma(F_i \boldsymbol{I}_i) \tag{13}$$

where, $f_0 = \partial p_0/\partial t + v_0 \Sigma(\partial p_j/\partial r_j) + \Sigma(b_j p_j + B_j P_j)$; $p_0 = \widehat{m} v_0$, $p_j = m v_j$; $P_i = M V_i$.

We have the octonion applied force density $\mathbb{F}'(f_0', f_1', f_2', f_3', F_0', F_1', F_2', F_3')$, when the coordinate system rotates. And then, we have the following result by Eq.(4).

$$f_0 = f_0' \tag{14}$$

When the right side is zero in the above, we have the mass continuity equation in the case for coexistence of the gravitational field and electromagnetic field.

$$\partial \widehat{m}/\partial t + \Sigma(\partial p_j/\partial r_j) + \Sigma(b_j p_j + B_j P_j)/v_0 = 0 \tag{15}$$

Further, if the strength is zero, $b_j = B_j = 0$, the above will be reduced as follows.

$$\partial m/\partial t + \Sigma(\partial p_j/\partial r_j) = 0 \tag{16}$$

The above states that the gravitational strength and electromagnetic strength have the influence on the mass continuity equation, although the $\Sigma(b_j p_j + B_j P_j)/v_0$ and the $\triangle m$ both are usually very tiny when the fields are weak. When we emphasize the definitions of applied force and velocity in gravitational and electromagnetic fields, the mass continuity equation will be the invariant equation under the octonion transformation.

7. CONCLUSION

The mass continuity equation will vary in the strong electromagnetic field or gravitational field, and has a deviation from the mass continuity equation described with the vectorial quantity. In the gravitational and electromagnetic fields, this states that the mass continuity equation will change with the electromagnetic field strength, gravitational field strength, linear momentum, electric current, and the speed of light. The deduction can explain why the field strength has an impact on the anomalous transport about the mass continuity equation in the plasma.

In the octonion space, the deductive results about the conservation laws and the invariants depend on the definition combinations in the case for coexistence of gravitational field and electromagnetic field. By means of the definition combination of the linear momentum, the quaternion operator, and the velocity, we have the conclusions about the mass continuity equation in the gravitational field and electromagnetic field.

It should be noted that the study for the mass continuity equation examined only one simple case of weak field strength. Despite its preliminary characteristics, this study can clearly indicate the mass continuity equation is an invariant and is only one simple inference due to the low velocity and the weak strengths of electromagnetic field and gravitational field. For the future studies, the investigation will concentrate on only some predictions about the mass continuity equation under the high speed and strong strength of electromagnetic field.

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